Discussion on Laboratory 1 Performance of a queuing system

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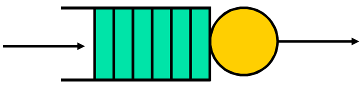
The M/M/1 queue

- The M/M/1 queue is characterized by:
  - Customer interarrival times distributed according to a negative exponential with rate λ
  - Customer service times distributed according to a negative exponential with rate μ
  - Single server
  - FIFO (first-in-first-out) service police
  - Waiting line with infinite capacity

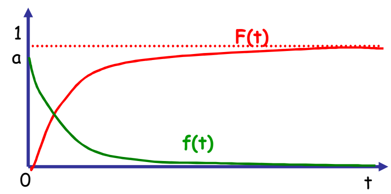
The M/M/1 queue



- On average λ customers arrive in the time unit
- The server serves on average μ customers per time unit
- The *load* is defined as λ/μ and coincides with the server utilization

Negative exponential

- Negative exponential distribution with parameter a, $t \geq 0$
  - $f(t) = ae^{-at}$, $F(t) = 1-e^{-at}$
  - $E[T]=1/a$, $C_v^2=1$ ($C_v^2=Var(T)/E[T]^2$)



Little's Law

- Given that the queue is stable

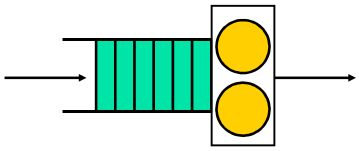
$$E[D] = \frac{E[N]}{\lambda}$$

where
- λ is the average number of customers arriving at the system in the time unit
- E[D] is the average time spent by a customer in the system
- E[N] is the average number of customers in the system

The two-server queue M/M/2

- The queue has two independent servers
- Servers are never idle if there is a customer waiting for service
- A customer that finds both servers available at its arrival at the queue, chooses randomly one of them

The two-server queue M/M/2



- On average λ customers arrive in the time unit
- Each server serves on average μ customers per time unit
- The *load* is defined as $\lambda/(2\mu)$

Task 1: System performance

The aim of task 1 is the extension of the simulator to a two-server queue M/M/2 and the evaluation of its performance

- The performance indexes to consider are:
  - Average number of customers in the system
  - Average delay for a customer

Task 1: System performance

1. Obtain simulation results for at least 5 values of the queue load between 0.5 and 0.95
2. Compare the results of the M/M/2 queue with service rate μ with those of the M/M/1 queue with service rate 2μ
3. Verify Little's Law

M/M/2

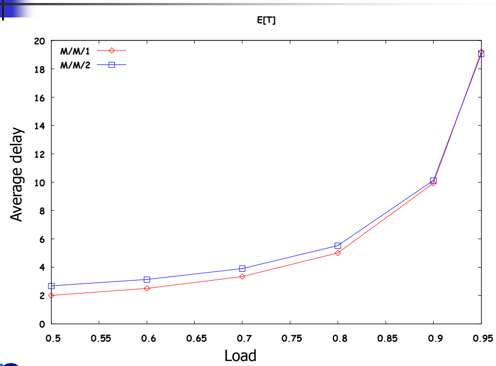
- From the M/M/2 analytical model, we know that

$$E[T] = \frac{1}{\mu} + \frac{\lambda^2}{\mu(4\mu^2 - \lambda^2)}$$

$$E[N] = \lambda E[T] = \frac{\lambda}{\mu} + \frac{\lambda^3}{\mu(4\mu^2 - \lambda^2)}$$

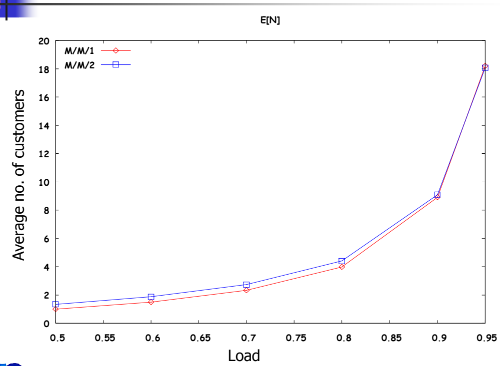
- We can use this information to validate the model

M/M/1 vs. M/M/2 [μ=0.5]



TNG

M/M/1 vs. M/M/2 [μ=0.5]



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Task 2: Limited Buffer Size

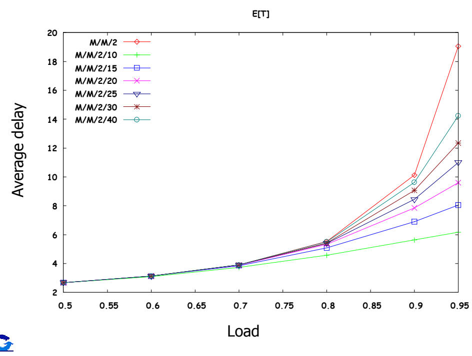
The aim of task 2 is the extension of the two-server queue simulator to a case with limited buffer size

- When at its arrival a customer finds B other customers already waiting, it cannot enter the queue and is *lost*
- As performance index, compute the customer loss probability

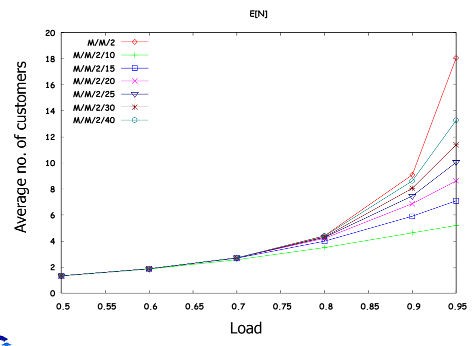
Task 2: Limited Buffer Size

1. Evaluate the queue performance (i.e. average delay, average number of customers and loss probability) for the same values of load considered in the previous task
2. For load equal to 0.9, evaluate loss probability as buffer size B increases considering some values in the range [10,40]
3. Find the smallest value of B that guarantees loss probability smaller than 1%

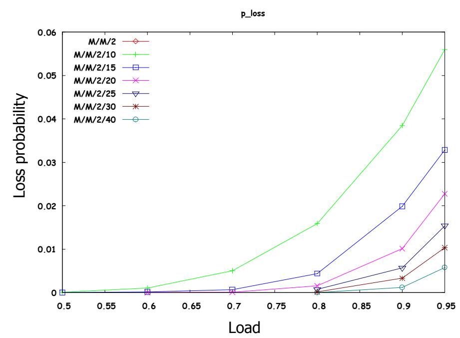
M/M/2/B [μ=0.5]



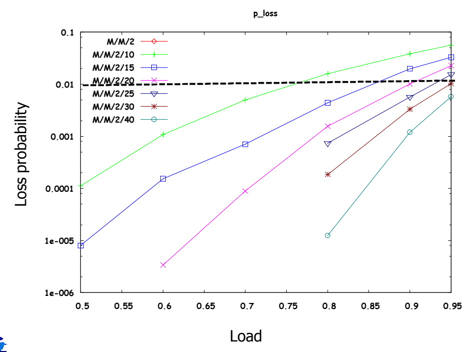
M/M/2/B [μ=0.5]



M/M/2/B [μ=0.5]



M/M/2/B [μ=0.5]



Task 3: The $M/H_2/2$ queue

The aim of task 3 is the extension of the two-server queue simulator to the queue $M/H_2/2$

- The $M/H_2/2$ differs from the M/M/2 in the service time that is distributed according to a 2-phase hyper-exponential distribution

Task 3: The $M/H_2/2$ queue

- The 2-phase hyper-exponential distribution:

$$f(t) = a\mu_1 e^{-\mu_1 t} + (1-a)\mu_2 e^{-\mu_2 t}$$

$$E[T] = a\frac{1}{\mu_1} + (1-a)\frac{1}{\mu_2}$$

$$E[T^2] = a\frac{2}{\mu_1^2} + (1-a)\frac{2}{\mu_2^2}$$

Task 3: The $M/H_2/2$ queue

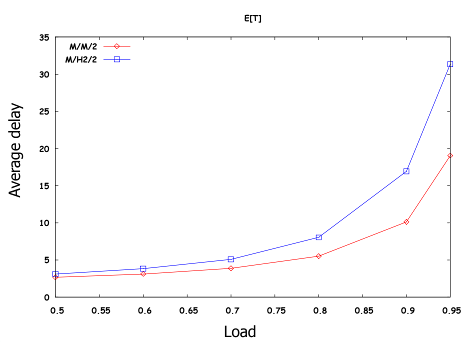
1. Evaluate performance (i.e., average delay, average number of customers) for the same values of load considered in the previous tasks
2. To compare the M/M/2 and $M/H_2/2$ systems, after having chosen the parameter μ for the M/M/2, choose the parameters for the $M/H_2/2$ as follows

$a=3/4$ $\mu_1=2\mu$ $\mu_2=0.4\mu$

The H_2 generator

- We need to implement a 2-phase hyper-exponential generator
- Observe that the pdf is already in a form suitable to apply the composition method
  1. Generate u_1=U(0,1) and u_2=U(0,1)
  2. If u_1<a, use u_2 to generate x from the neg. exponential with rate μ_1 -> $x = -\ln(u_2)/\mu_1$
  3. Otherwise, use u_2 to generate x from the neg. exponential with rate μ_2 -> $x = -\ln(u_2)/\mu_2$

M/M/2 vs. $M/H_2/2$ [C_v^2=2.5]



M/M/2 vs. $M/H_2/2$ [C_v^2=2.5]